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ABSTRACT

The first of four "quins" designed to strengthen fundamental concepts and skills, this course covers properties of real numbers, simple open sentences, factorization of natural numbers, and problem solving. After a list of overall goals, the guide gives performance objectives, course outline, references to state-adopted textbooks, and suggested strategies for four units. Also included is a sample pretest, a sample posttest, and a student bibliography. (MM)

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AUTHORIZED COURSE OF INSTRUCTION FOR THE QUINMESTER PROGRAM

MATHEMATICS

PRE-ALGEBRA 1

5210.11

DADE COUNTY PUBLIC SCHOOLS

DIVISION OF INSTRUCTION • 1971

QUINMESTER MATHEMATICS

COURSE OF STUDY

FOR

PRE-ALGEBRA 1

5210.11

(EXPERIMENTAL)

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1971-72

PREFACE

The following course of study has been designed to set a minimum standard for student performance after exposure to the material described and to specify sources which can be the basis for the planning of daily activities by the teacher. There has been no attempt to prescribe teaching strategies; those strategies listed are merely suggestions which have proved successful at some time for some class.

The course sequence is suggested for a guide; an individual teacher should feel free to rearrange the sequence whenever other alternatives seem more desirable. Since the course content represents a minimum, a teacher should feel free to add to the content specified.

Any comments and/or suggestions which will help to improve the existing curriculum will be appreciated. Please direct your remarks to the Consultant for Mathematics.

All courses of study have been edited by a subcommittee of the 1970-71 Mathematics Advisory Committee.

CATALOGUE DESCRIPTION

The first of four quins designed to strengthen fundamental concepts and skills which are basic preparation for Algebra 1. Includes properties of real numbers, simple open sentences, factorization of natural numbers, and problem solving.

Designed for the student who has mastered the fundamental computational skills with rational numbers.

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OVERALL GOALS

1. To further develop student understanding of the commutative, associative, and identity properties for addition and multiplication of non-negative rational numbers.

2. To improve student skill in computation with non-negative rational numbers.

3. To enable the student to understand and use the rules of divisibility in prime factorization; and to find the greatest common factor and least common multiple.

4. To familiarize student with the use of variables in simple open sentences.

5. To assist student in gaining insight in solving verbal problems.

KEY TO STATE ADOPTED REFERENCES

D - Dodes, Irving A. Mathematics: A Liberal Arts Approach. New York: Hayden Book Co., Inc., 1964.

E(I) - Eicholz, Robert; O'Daffer, Phares; Brumfiel, Charles; Shanks, Merrill; Fleenor, Charles. School Mathematics I. Menlo Park, California: Addison-Wesley Publishing Co., 1967.

E(II) - Eicholz, Robert; O'Daffer, Phares; Brumfiel, Charles; Shanks, Merrill; Fleenor, Charles. School Mathematics II. Menlo Park, California: Addison-Wesley Publishing Co., 1967.

K(1) - Keedy, Mervin; Jameson, Richard; and Johnson, Patricia. Exploring Modern Mathematics, Book 1. New York: Holt, Rinehart and Winston, Inc., 1963.

Mc(7) - McSwain, E. T.; Brown, K. W.; Gundlach, B. H.; and Cooke, R. J. Mathematics 7. River Forest, Illinois: Laidlaw Brothers, 1965.

Mc(8) - McSwain, E. T.; Brown, K. W.; Gundlach, B. H.; and Cooke, R. J. Mathematics 8. River Forest, Illinois: Laidlaw Brothers, 1965.

N - Nichols, Eugene D. Pre Algebra Mathematics. New York: Holt, Rinehart and Winston, Inc., 1965.

SW - Skeen, Kenneth C. and Whitmore, Edward H. Modern Math, Book I. Atlanta: L. W. Singer Co., 1966.

SM - Sobel, Max A. and Maletsky, Evan M. Essentials of Mathematics, Skills and Concepts 3. Upper Montclair, New Jersey: Ginn and Company, 1969.

I. NON-NEGATIVE RATIONAL NUMBERS

Performance Objectives

The student will

1. Recognize the commutative, associative, and identity properties with respect to addition and multiplication of rational numbers, when illustrated by a specific example.

2. Make accurate use of the properties named above.

3. Perform the basic arithmetic operations with any given pair of positive rational numbers.

Course Outline

I. Non-negative rational numbers
   A. Commutative property
      1. Definition
      2. Addition
      3. Multiplication
   B. Associative property
      1. Definition
      2. Addition
      3. Multiplication
   C. Identity property
      1. Definition
      2. Addition
      3. Multiplication
   D. Skill practice
      1. Addition
      2. Subtraction
      3. Multiplication
      4. Division
      5. Equivalents

State Adopted References

D (pp. 34-50 and 70-94) provides for discovery then states the property; not many exercises for reinforcement.

E(I) (pp. 39-52 and 139-148) states the principle, then uses number sentence exercises for reinforcement.

K(1) (pp. 57-70, 194-202,231, 285) uses the discovery approach to introduce the properties of whole numbers.

Mc(7) (pp. 21-24, 99-101, 104-144) explains the properties, then gives exercises to reinforce.

N (pp. 99-104, 132-145, 239-242) explains as in Mc(7) but structures the same as K(1).

SW (pp. 36-39, 42-45, 111-176) gives an example, explains the property for natural numbers, then states that it holds for rational numbers; not many practice exercises for properties but good review of operations with positive rationals.

SM (pp. 133-137) same approach as SW but more practice examples with integers; has a programmed review of both decimals and fractions; identities not covered.

Suggested Strategies

1. The student should know that A, B, and C are properties of a set of numbers with respect to an operation, not to be confused with properties of the relations: equals, less than, is parallel to, is congruent to, etc.

2. To clarify the commutative properties, have the student specify whether it is the addends or the factors which are being commuted; for example,

$$(5 + 3) \times 2 = 2 \times (5 + 3)$$
$$\text{and } (5 + 3) \times 2 = (3 + 5) \times 2$$

3. Take sufficient time to firmly reinforce the four operations with positive rationals.

Student Reference

Dressler, Isidore. Preliminary Mathematics. New York: Amsco School Publications, Inc., 1965.

II. FACTORS AND PRIMES

Performance Objectives

The student will

1. Use the rules of divisibility for 2,3,4,5,6,9,10,25 to test divisibility of natural numbers.

2. Define a prime number.

3. Determine the factors of any natural number.

4. Find the prime factorization of any natural number.

5. Define the greatest common factor.

6. Find the greatest common factor of any two natural numbers.

7. Exhibit the set of multiples of any natural number.

8. Determine the least common multiple of any two natural numbers.

Course Outline

II. Factors and Primes
- A. Divisibility
  - 1. Rules for 2,3,4,5,6, 9,10,25
  - 2. Application to prime factorization
- B. Factorization of natural numbers
  - 1. Primes
    - a. Definition
    - b. Finding primes
  - 2. Composites
    - a. Definition
    - b. Non-prime factorization
    - c. Prime factorization
- C. Greatest common factor
  - 1. Definition of GCF
  - 2. GCF of two or more natural numbers
- D. Multiples
  - 1. Determination of multiples of natural numbers
  - 2. Definition of least common multiple (LCM)
  - 3. LCM of two or more natural numbers

State Adopted References

E(I) (pp. 137-151) does not use divisibility, but uses the listing of factors and multiples to find GCF and LCM.

K(1) (pp. 135-184) uses prime factorization to find LCM and GCF.

Mc(7) (p. 83) develops divisibility, prime numbers, GCF, and then LCM.

N (pp. 67-89) uses the intersection of sets for LCM and GCF.

SW (pp. 111-153) no divisibility; same approach as E(I).

SM no divisibility, no prime numbers, only talks about least common denominator; (p. 195) just mentions greatest common factor, assumes students know it.

II. Factors and Primes (continued)

Suggested Strategies

1. Use Sieve of Eratosthenes to exhibit primes while reinforcing understanding of multiples.

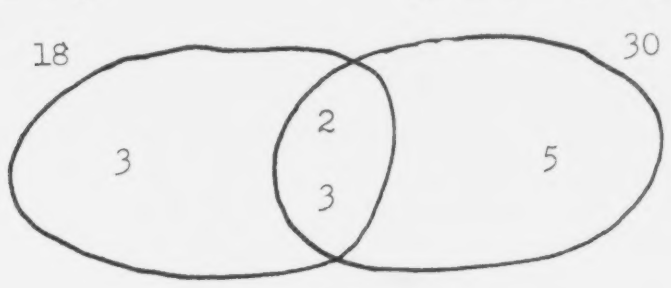
2. Stress unique prime factorization of natural numbers.

3. Point out the special status of the natural number 1; i.e., one is neither a prime nor a composite.

4. Relate 3 to unique factorization as follows:
   If 1 were a prime, then $1^3 \cdot 2 = 1^2 \cdot 2 = 1 \cdot 2$ and there would not be a unique prime factorization of 2.

5. Use Venn diagrams to find GCF and LCM as follows:

$18 = 2 \cdot 3^2$ $\qquad$ $30 = 2 \cdot 3 \cdot 5$



union $= 3 \cdot 3 \cdot 2 \cdot 5 =$ LCM
intersection $= 2 \cdot 3 =$ GCF

This interesting strategy is good for students at this level but becomes too sophisticated when three numbers are involved.

Student Reference

Weber, Rose and Weber, Ruth. The New Mathematics 8. Wichita, Kansas: McCormick-Mathers Publishing Co., 1964.
Excellent approach to all areas covered in the outline.

III. SIMPLE EQUATIONS

Performance Objectives

The student will

Intuitively solve equations of the form $ax = b$ and $a + x = b$ where a and b are non-negative integers.

Course Outline

III. Simple Equations
- A. Use of variables
- B. Intuitive solutions to equations of the form
  1. $ax = b$
  2. $a + x = b$

  where a and b are non-negative integers.

State Adopted References

E(I) (pp. 36-41) uses intuitive approach but also brings in the idea of inverse relationships.

K(1) (pp. 50-52) defines "equations" and "variables"; also used symbols other than letters as variables in equations.

Mc(7) (pp. 33-34) uses the axioms of equality to introduce solving simple open sentences.

N (pp. 403-409) has a definition for "variable" and explains the "cover-up" method of solving sentences.

SW (pp. 161-167 and 264-271) same as Mc(7).

Suggested Strategies

1. Use substitution as a means for checking intuitive solutions.
2. Use integral coefficients.

Student Reference

Wohlfort, Sheridan. Investigating Mathematical Ideas: Skillbooks B and D. New York: Holt, Rinehart, Winston, Inc., 1969.

IV. PROBLEM SOLVING

Performance Objectives

The student will

1. Translate simple word phrases and sentences into symbols.

2. Demonstrate his understanding of algebraic notation by creating a verbal problem to fit a given open sentence.

3. Use the flow chart method to solve problems.

Course Outline

IV. Problem Solving
   A. Translation from words to symbols
   B. Creation of verbal problems
   C. Use of flow charts in problem analysis

State Adopted References

Mc(7) (p. 37) has some good background ideas.

Mc(8) (pp. 85-87) has good treatment of translating words to symbols.

N (pp. 404-408) has useful information and numerous examples of translating words to symbols.

SW (pp. 167-169 and 271-272) plenty of practice but no explanation.

SM (pp. 320-352) starts with the flow chart approach then moves into intuitive translating from words to symbols.

Suggested Strategies

Investigate flowcharting as a technique for problem solving. There is an article in "The Mathematics Teacher", April, 1971, which explains the application to problem solving.

Student Reference

Weber, Rose and Weber, Ruth. The New Mathematics 7 and 8. Wichita, Kansas: McCormick-Mathers Publishing Co., 1964.

SAMPLE PRETEST

To make the results of the pretest more meaningful, it is recommended that there be one day of practice prior to the test.

I. Multiple choice:
Place the letter of the correct response in the space provided.

1. $\frac{3}{5} + \frac{10}{11}$

a. $\frac{30}{55}$ b. $\frac{13}{16}$ c. $\frac{83}{55}$ d. $1\frac{25}{30}$ e. none of these f. all of these

2. $\frac{2}{3} \times \frac{8}{10}$

a. $\frac{8}{15}$ b. $\frac{5}{6}$ c. $\frac{20}{24}$ d. $\frac{10}{13}$ e. none of these f. all of these

3. $\frac{8}{9} \div \frac{4}{10}$

a. $\frac{36}{80}$ b. $\frac{2}{9}$ c. $\frac{12}{19}$ d. $\frac{2}{4.5}$ e. none of these f. all of these

4. $7\frac{1}{8} - 3\frac{1}{6}$

a. $4\frac{1}{24}$ b. $3\frac{23}{24}$ c. $4\frac{23}{24}$ d. $4\frac{1}{2}$ e. none of these f. all of these

II. Complete each of the following in the space provided:

5. $306 - 1.5 =$ _____

6. Give an example of the commutative property for addition.

7-8. _____ is a factor of _____.

9-10. _____is a multiple of _____.

11. Find the set of numbers which are divisors of the numeral "two hundred thirty-six million, seven hundred eighty-four thousand, one hundred thirty-five."

12. The prime factorization of 120 is _____.

Pretest (continued)

13. The greatest common factor of 24 and 36 is _____.

14. The least common multiple of 24 and 36 is _____.

15. Find n, if n + 8 = 13. _____.

16. Find n, if 3n = 27. _____.

17. Find n, if 3n = 7. _____.

18. Is 51 a composite number? Defend your answer.

SAMPLE POSTTEST

1. Give an illustration of each of the following properties of rational numbers:

   a. Commutative property for addition

   b. Commutative property for multiplication

   c. Associative property for addition

   d. Associative property for multiplication

   e. The identity property for addition

   f. The identity property for multiplication

2. In each case, identify by complete name or accepted abbreviation the property illustrated:

   a. $7 \times 8 = 8 \times 7$

   b. $a \cdot 1 = a$

   c. $(a + b) + c = a + (b + c)$

   d. $\frac{1}{2} + \frac{2}{3} = \frac{2}{3} + \frac{1}{2}$

   e. $9 + 0 = 9$

   f. $(a \times b) \times c = a \times (b \times c)$

3. Perform the operations as indicated. All results must be reduced to lowest terms.

   a. $\frac{10}{20} - \frac{2}{20} =$

   b. $1\frac{1}{2} - 1\frac{1}{4} =$

   c. $4.2 - 1.3 =$

   d. $100 - .75 =$

   e. $\frac{3}{7} - \frac{2}{21} =$

   f. $\frac{3}{4} \times \frac{5}{5} =$

   g. $6 - \frac{3}{4} =$

   h. $\frac{1}{2} \times \frac{1}{2} =$

   i. $29 \times .01 =$

   j. $4.2 \times .06 =$

   k. $\frac{2}{3} \div \frac{4}{5} =$

   l. $6.4 \div .2 =$

   m. $\frac{4}{8} = \frac{?}{10}$

   n. $\frac{1}{10} \div 3 =$

   o. $\frac{4}{25} = \frac{16}{?}$

   p. $\frac{1}{8} - \frac{1}{2}$

   q. $6.2 - .043 =$

Posttest (continued)

4. Identify whether each of the following is true or false by circling T or F:

T F a. $\frac{6}{3} < \frac{16}{4}$

T F b. $\frac{15}{3} = \frac{20}{5}$

T F c. $\frac{12}{16} = \frac{6}{9}$

T F d. $\frac{3}{4} > \frac{74}{100}$

T F e. $1.407 > 2.1$

T F f. $16 < 3.748$

5. Using rules of divisibility, circle every number on the right which is a divisor of the given natural number.

a. 372	2, 3, 4, 5, 6, 8, 9, 10
b. 3670	2, 3, 4, 5, 6, 8, 9, 10
c. 495	2, 3, 4, 5, 6, 8, 9, 10
d. 840	2, 3, 4, 5, 6, 8, 9, 10

6. Circle each prime number in the set below:

1, 5, 7, 8, 9, 11, 17, 18, 21, 27, 87, 92, 225

7. List all different factors for each given number.

a. 108 b. 105

8. Give the prime factorization for each:

a. 108 b. 1540

9. Find the greatest common factor for each pair of natural numbers:

a. 48, 120 b. 105, 150

10. Determine the least common multiple for each pair of natural numbers:

a. 15, 25 b. 36, 48

Posttest (continued)

11. Determine the replacement for the variable which will make each statement true.

    a. $n + 2 = 7$

    b. $3c - 4 = 8$

    c. $5x = 25$

    d. $7d + 6 = 27$

12. Using numerals and variables express the following as number phrases or sentences:

    a. Some number plus six equals fourteen.

    b. Eight less than a number.

    c. Three times a number is thirty.

    d. Two more than twice a number.

13. Make a flow chart in solving these problems:

    a. John is 3 times as old as Mary. If Mary is 8 years old, how old is John?

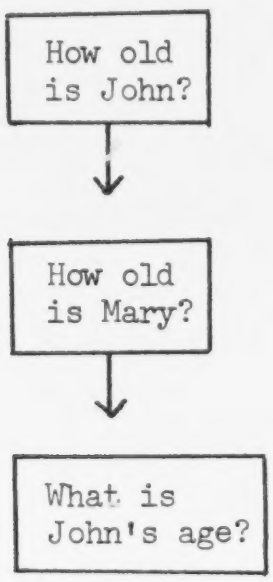
    b. Six more than twice a number is 20. What is the number?

    c. The length of a rectangle is 4 feet less than twice the width. If the length is 18 feet, what is the width?

XXXXXX

Note to the teacher: Following are some possible solutions using flow charts.

13. a.



13. b.

Name the number	n
What is twice the number?	2n
6 more than twice the number	6 + 2n
20 = ?	20 = 6 + 2n
2n = ?	2n = 14
n = ?	n = 7

13. c.

Name the length	ℓ
Name the width	w
Twice the width	2w
4 ft. less than twice the width	2w - 4
ℓ = ?	ℓ = 2w - 4
But ℓ = 18, so 18 = ?	18 = 2w - 4
2w = ?	2w = 22
w = ?	w = 11

STUDENT BIBLIOGRAPHY

1. Bechenbach, Edwin and Bellman, Richard. An Introduction to Inequalities. New Mathematical Library. New York: Random House, Inc., 1963.

2. Dressler, Isidore. Preliminary Mathematics. New York: Amsco School Publications, Inc., 1965.

3. Eason, Oliver W. Graphing Pictures. Portland, Maine: J. Weston Walsh, 1966.

4. Estes, Ronald V. and Kerber, Karl. The New Mathematics: Algebra. Wichita, Kansas: McCormick-Mathers Publishing Co., 1966.

5. Keedy, Mervin; Jameson, Richard; and Johnson, Patricia. Exploring Modern Mathematics, Book 1. New York: Holt, Rinehart and Winston, Inc., 1963.

6. L S I Contemporary Mathematics Kit. California Test Bureau. Monterey, California: McGraw-Hill Book Co.

7. Niven, Ivan. Numbers: Rational and Irrational. New Mathematical Library. New York: Random House, 1961.

8. Ore, Oystein. Graphs and their Uses. New Mathematical Library. New York: Random House, 1963.

9. Weber, Rose and Weber, Ruth. The New Mathematics 7. Wichita, Kansas: McCormick-Mathers Publishing Co., 1964.

10. Weber, Rose and Weber, Ruth. The New Mathematics 8. Wichita, Kansas: McCormick-Mathers Publishing Co., 1964.

11. Wohlfort, Sheridan. Investigating Mathematical Ideas: Skillbook B and D. New York: Holt, Rinehart, and Winston, Inc., 1969.